On new concepts effecting guidance law synthesis of future interceptor missiles

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NOUVEAUX CONCEPTS AFFECTANT LA SYNTHESE DE LOIS DE GUIDAGE POUR DES MISSILES FUTURS

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RÉSUMÉ. Alors que la technologie des missiles d'interception a fait des progrés considérables depuis une trentaine d'années, les lois de guidage sont réstées très classiques. Nous introduisons ici de nouvelles lois de guidage largement fondées sur la théorie des jeux différentiels, et nous discuttons leur impact potentiel sur les trajectoires.

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ABSTRACT. Though the technology of interceptor missiles has made an impressive progress during the last three decades, guidance law synthesis has remained conservative. In the present paper new guidance concepts, derived mainly by differential game analysis, are introduced and their potential impact on future synthesis of homing strategies is discussed. Adoption of these new concepts will enhance missile performance by yielding enlarged effective firing envelopes as well as improved homing accuracy.

1. INTRODUCTION.

Guided missile performance can be measured by two closely related criteria: terminal accuracy and the effective firing zone where this accuracy can be attained. The guidance accuracy depends on the quality of target detection and on the guidance law which translates the information on target position to a control of the trajectory for attaining this target. Repeating this definition in more precise terms, one can say that missile guidance law is a mathematical expression (a mapping) which relates the set of available measurments to the commanded lateral acceleration of the missile. The effective firing zone is determined, for a given set ot initial conditions, mainly by the amount of propulsive energy stored in the missile and the aerodynamic characteristics of its airframe. Nevertheless, for an interceptor missile this zone is also a function, – a rather sensitive one, – of the target motion. Since trajectory corrections require additional energy dissipation, the effective firing zone of the missile depends furthermore on the guidance law.

In the first generations of homing interceptor missiles the guidance law of Proportional Navigation (PN) have been used, almost without exeption, mainly for two important reasons: (i) This guidance law, conceived during the second world war and published (after declassification) several years later [1], generates a trajectory which tends asymptotically to an ideal collision course. (ii) Moreover, the implementation of PN can be performed using angular information only.

By assuming trajectory linearization around the nominal collision course, as well as constant velocities and instantenous missile dynamics, the equations of motion can be solved in a closed form [2-5]. This analytic solution has provided a basis for trajectory analysis and for setting design requirements. Taking into account linear, time-invariant, but otherwise realistic guidance channel dynamics resulted in predicting unbounded terminal accelerations. However, it was shown that the miss distances, – generated by launching errors, by constant target maneuvers, as well as by a reasonable amount of measurment or system noise, – can be kept in the range of admissible values for a successfull operation by the available lateral acceleration limit of a well designed missile.

The linearized mathematical model used for the analysis imbedded the assumptions of a constant closing speed and a fixed flight time. As a consequence, it was unable to deal with firing zone considerations. Firing zone computation were generally carried out using point-mass equations of motion with relevant propulsion and aerodynamic models and assuming a horizontally flying constant speed target.

Operational experience has shown that the missiles of the first generations could not be used successfully against smartly maneuvering targets. Studies of optimal evasion from guided missiles [6-12] have indeed demonstrated that the knowledge of guidance law parameters can be exploited for determining an optimal evasive maneuver and achieving a rather large miss distance. Moreover, firing zone computations based on the assumption of straight flight are invalidated by any significant target maneuvering.

Since the time when optimal control has became fashionable, the problem of missile guidance law optimization has served as an interesting example [13-17]. These studies, which used the analytically convenient linearized trajectory model and a quadratic cost function, indicated that by including information on target maneuver and missile dynamics in the guidance law an improved performance (compared to PN) can be achieved. The practical application of guidance law optimization has been rather limited, because missile designers discovered [18] that such "modern" guidance laws are sensitive to modelling errors and other uncertainties. Consequently, they are less robust than PN. The greatest uncertainty of all is the actual target maneuver, which is generally unknown, mostly unpredictable and hardly measurable. Based on this observation one must reach the conclusion that the *optimal control* formulation of the interceptor missile guidance problem is uncomplete. In this problem there are two independent controllers and as a consequence it has to be formulated as a *differential game*.

The objective of the present paper is to review several recent studies investigating the missile-aircraft interception as a zero-sum pursuit-evasion differential game and to point out those results that may effect future missile designs. In such designs optimal target maneuvering (in a differential game sense) must be explicitly considered in order to guarantee robustness against any type of possible target behavior.

In the next section the missile-aircraft interception problem is described and formulated as a differential game. In the sequel two different models are introduced. The first is a nonlinear point-mass dynamics model, appropriate for firing zone computations. The other, which is adaquate only for an end-game analysis, is based on the assumptions of linearized trajectory and constant speeds but includes missile dynamics. For each model optimal guidance laws are presented and their implementation in future designs is discussed.

2. PROBLEM STATEMENT.

Interception of a moving target such as an aircraft by a guided missile, launched either from a fixed or from a moving platform, is a complex dynamic problem composed of several phases. Once the target is detected, the launching platform is oriented towards a favorable initial condition for the missile. Simultaneously, a local firing computer estimates when to launch the missile in order to reach the target.

The missile trajectory itself can be devided into three major segments. In the first one the missile is oriented towards a future interception point. This phase becomes extremely important whenever the launching platform fails to tho perform this task, which may often happen in a short-range air combat. The next segment, called the "midcourse", is devoted to reach the vicinity of the target with sufficient speed in order to garantee a potential for success in the terminal phase. This is during the "end-game" where the success of the missile, measured generally by the smallness of the miss distance, is determined.

In each trajectory segment the major objective is different. The first segment is dedicated to the correction of evantual launching errors. The objective of the second phase is efficient energy management, while in the "end-game" the emphasis is on accuracy.

For short-range missile trajectories the "midcourse" may disappear and the energy management becomes an additional and sometimes conflicting objective in the inital phase. For medium-range missiles (as well as for those designed for long-range interceptions) the importance of the initial segment decreases with increasing range and the main emphasise is on efficient "midcourse" guidance.

Target maneuvering may have different effect on interceptor missiles, depending on their flight time. The relative geometry cannot change much during the engagement with a very short range missile. If, however, the effective missile flight time is long enough to anable the target to perform a large turn, the relative geometry can be entirely changed. In such a case the target maneuver has the potential to modify substantially the *effective* firing range of the missile. Moreover, target maneuvering in the "end-game" has a great effect on the accuracy of all missiles.

As already noted in the Inroduction, target maneuvering is unpredictable and hardly measurable either by the missile or by the launching platform. Therefore, it has to be considered as an independently controlled variable. As a consequence, in this paper the missile-aircraft interception is formulated as a pursuit-evasion differential game [19]. In this game the roles of the players are clearly defined: the missile is the pursuer and the aircraft is the evader.

Let $\mathbf{P}$ and $\mathbf{E}$ be the respective position vectors of the pursuer and the evader in any inertial coordinate system. Thus the range vector $\mathbf{R}$ can be defined as

$$\mathbf{R} = \mathbf{E} - \mathbf{P} \tag{1}$$

The the equations of the relative motion are

$$\dot{\mathbf{R}} = \mathbf{V}_E - \mathbf{V}_P \tag{2}$$

$$\dot{\mathbf{V}}_E = \mathbf{A}_E \tag{3}$$

$$\dot{\mathbf{V}}_P = \mathbf{A}_P \tag{4}$$

The forces $\mathbf{F_E}$ and $\mathbf{F_P}$, which produce the accelerations $\mathbf{A}_E$ and $\mathbf{A}_P$, are nonlinear functions of the respective position and velocity but they are also governed by a vector of internal variables $\mathbf{B}$, such as angle of attack, bank angle or engine rpm, etc.

$$m_E\mathbf{A}_E = \mathbf{F}_E(\mathbf{E}, \mathbf{V}_E, \mathbf{B}_E) \tag{5}$$

$$m_P\mathbf{A}_P = \mathbf{F}_P(\mathbf{P}, \mathbf{V}_P, \mathbf{B}_P) \tag{6}$$

(m_E, m_P are the masses of the evader and the pursuer respectively.)

The internal variables have their own dynamics expressed by

$$\dot{\mathbf{B}}_E = \mathbf{G}_E(\mathbf{E}, \mathbf{V}_E, \mathbf{B}_E, \mathbf{u}_E) \tag{7}$$

$$\dot{\mathbf{B}}_P = \mathbf{G}_P(\mathbf{P}, \mathbf{V}_P, \mathbf{B}_P, \mathbf{u}_P) \tag{8}$$

The vectors $\mathbf{u}_E, \mathbf{u}_P$ are the controls of the evader and the pursuer respectively. All control vector components (being of physical nature) are bounded i.e. the control vectors belong to compact sets

$$\mathbf{u}_E \in U_E \tag{9}$$

$$\mathbf{u}_P \in U_P \tag{10}$$

Eqs. (2)-(8) can be summarized in a single nonlinear autonomous vector equation representing the entire game dynamics

$$\dot{\mathbf{X}} = \mathcal{F}(\mathbf{X}, \mathbf{u}_E, \mathbf{u}_P), \qquad \mathbf{X}(t_0) = \mathbf{X}_0 \tag{11}$$

defined in a domain $\mathcal{D}$ of the game space $\mathcal{R}^n$, where $\mathbf{X}$ is a high dimensional state vector of the game.

$$\mathbf{X} = (\mathbf{E}, \mathbf{P}, \mathbf{V}_E, \mathbf{V}_P, \mathbf{B}_E, \mathbf{B}_P)^T \in \mathcal{R}^n \tag{12}$$

A differential game formulation requires to determine, in addition to the game dynamics and its domain of validity $\mathcal{D}$, (i) the information structure; (ii) the criterion for game termination; and (iii) the cost function of the game. Since any small variation in these elements can substantially modify the game solution, a generalized formulation (comparable to the description of the dynamics) cannot be given.

As already mentioned in the Introduction, the performance assessment of an interceptor missile is generally carried out in two diffetent steps: the determination of the effective firing envelope and the evaluation of guidance accuracy inside this envelope. One of the reasons for this separation is the inherent complexity of game dynamics presented in Eqs.(2)-(10). In the present paper a similar approach is followed, leading to formulate two distinct pursuit-evasion games. In each game a different approximation of the dynamics is used. In one game the emphasis is on the first two segments of the missile trajectory, while the other concentrates on "end-game" analysis.

3. THE "FIRING ENVELOPE" GAME.

In this game the objective of the missile, as it is stated in the previous section, *is to reach the vicinity of the target with sufficient speed in order to guarantte a potential for success in the terminal phase.* This statement implies that the essential elements of the problem are the nonlinear geometry and the speed variations of missile. This observation allows to neglect the dynamics of the internal variables described in Eqs. (7) and (8).

Since firing envelope is a deterministic concept, in the present game *perfect information* is assumed. This assumption means that problem parameters are known and all state variables are accurately measured by both players.

The termination of this pursuit-evasion game is generally determined by a set of inequalities.

$$|\mathbf{V}_P(t_f)| \geq (V_P)_{min} \tag{13}$$

$$|\dot{\mathbf{R}}(t_f)| \geq (V_c)_{min} \tag{14}$$

$$|\mathbf{R}(t_f)| \geq R_f \tag{15}$$

$$t_f \leq T \tag{16}$$

The game terminates whenever any of these four inequalities is violated. The terminal manifold of the game can be thus expressed as

$$\mathbf{\Psi}[\mathbf{X}(t_f), t_f] = 0 \tag{17}$$

The game terminates "successfully" for the missile (pursuer) if inequality (15) is the one which is violated first. By definition, *the effective firing envelope* is the boundary of the set of all initial states $\mathbf{X}_0$ which satisfy the above stated condition of missile success. The problem can be formulated in this case as a *pursuit-evasion game of kind* [19].

In such game formulation there is no specific cost function. The analysis is rather oriented to determine the "capture zone" of the game. The objective of the pursuer is to maximize this zone and the evader wishes to minimize it. Both players search for the respective optimal strategies in order to achieve their conflicting objectives.

The dynamic model of the game, even after neglecting Eqs. (7) and (8), is too complex to yield an analytical solution. A few years ago an attempt was made to approximate the optimal evader strategy [20,21] by using a singular perturbation approach. One of the conclusions of the study, oriented to investigate the *"no-escape" firing envelopes* (the "capture zones") of several missiles [21], was that Proportional Navigation is far from being optimal for this purpose.

Recently a simplified dynamic model, assuming a horizontal engagement between a constant speed evader and a coasting pursuer, was successfully analysed. The results of this study, reported in [22-25], are briefly summarized in the sequel. The variables of the planar geometry are defined in Fig. 1. The equations of motion in line of sight coordinates are:

$$\dot{R} = -(V_E \cos\phi_E + V_P \cos\phi_P) \tag{18}$$

$$\dot{\phi}_E = \Gamma_E u_E - \dot{\Theta}; \qquad |u_E| \leq 1 \tag{19}$$

$$\dot{\phi}_P = \Gamma_P u_P - \dot{\Theta}; \qquad |u_P| \leq 1 \tag{20}$$

where Γ_E and Γ_P are the maximum turning rates of the players and

$$\dot{\Theta} = -\frac{V_E \sin\phi_E + V_P \sin\phi_P}{R} \tag{21}$$

The speed dynamics of the pursuer (missile) can be written as

$$\dot{V}_P = -V_P{}^2(A + B u_P{}^2) \tag{22}$$

where A and B are (using standard aerodynamical notations):

$$A = \frac{0.5\rho S}{m} C_{D_0} \tag{23}$$

$$B = \frac{0.5\rho S}{m} K C_{L_{max}}{}^2 \tag{24}$$

In the present game of kind the inequalities (13) and (15) are transformed to equalities leaving the final values of ϕ_E and ϕ_P free. Fortunately, the costate equations of this game can be integrated in a closed form [22] by using the set of non dimensional variables:

$$v = \frac{V_P}{V_E} \tag{25}$$

$$r = \frac{R}{R_{ref}} \tag{26}$$

$$\bar{t} = \frac{t}{t_{ref}} = \frac{tV_E}{R_{ref}} \tag{27}$$

with R_{ref} being the minimum turning radius of the pursuer,

$$R_{ref} = \frac{m}{0.5\rho S C_{L_{max}}} \tag{28}$$

and the ratio of the minimum turning radii of the pursuer and of the evader being denoted by

$$\sigma = \frac{R_{ref}\Gamma_E}{V_E} \tag{29}$$

The analytical form of the costate solution allows to express the optimal strategies of the players as explicit functions of the current and final state.

The optimal strategy of the evader (the target aircraft) is simply a turn towards the final line of sight direction.

$$u_E{}^* = sign\left([\cos\phi_{Ef} - \cos(\phi_E + \Theta - \Theta_f)][\sin(\phi_E + \Theta - \Theta_f)]\right) \tag{30}$$

If this direction is reached, the target continues to fly in a straight line ($u_E{}^* = 0$) along a singular "universal" surface of the game.

The optimal strategy of the pursuer, which can be interpreted as the guidance law of the missile, is given by

$$u_P{}^* = \frac{N}{D} \tag{31}$$

with

$$N = r\sin(\Theta - \Theta_f) + \frac{[\cos\phi_{Ef} - \cos(\phi_E + \Theta - \Theta_f)]sign[\sin(\phi_E + \Theta - \Theta_f)]}{\sigma} \tag{32}$$

and

$$D = -2KC_{Lmax}\cos\phi_{Ef}\left[\tau + \frac{C_{L_{max}}}{C_{D_0}}\left(\frac{1}{V_f{}^*} - \frac{1}{V_f}\right)\right] \tag{33}$$

where τ is the normalized time-to-go (it can be expressed also as an explicit function of the current and terminal state variables) and

$$V_f{}^* = \frac{\cos\phi_{Ef}}{\cos\phi_{Pf}} \tag{34}$$

It is easy to see that the missile is turning, similarly to the target, towards the final line of sight direction and that the optimal missile maneuver is composed of two parts. One part compensates for the line of sight rotation, while the other reacts to the optimal target maneuver. Note also, that the denominator is proportional to the induced drag parameter of the missile. These features result in an optimal energy management for the missile. The optimal maneuver has an interesting geometrical representation depicted in Fig.2.

Though this control strategy cannot be implemented directly, because it depends on the unknown final state variables, it can be well approximated by a *closed-loop* guidance law [24,25] and can serve as a basis for comparison. A recently completed study [26] demonstrates the great advantage of the *game optimal* missile guidance law compared to Proportional Navigation. In Fig. 3 the "capture zones" (the *"no-escape" firing envelopes*) of the same missile with two different guidance laws are depicted for a set of very favorable initial conditions for the missile ($\phi_{P0} = 0$ i.e. "boresight" firing). Here the advantage is about 10-20% in the initial range. For initial conditions unfavorable to the missile the difference can be very large. In one of the examples presented in [25] (with $\phi_{P0} = 10^o$, $\phi_{E0} = -90^o$), the difference amounted to 77.45%. Moreover, there are many unfavorable initial conditions from which a PN guided missile is unable to reach the target, while the optimal guidance law scores "capture". The results of this comparison clearly indicate the potential performance improvement of a future "dogfight missile" using a *game optimal* guidance law.

4. "END-GAME" ANALYSIS.

4.1. Linearized "End-Game" Dynamics.

The "end-game" of a missile-target encounter is characterized by a short duration and a relative geometry, which justify a trajectory linearization with respect to a nominal collision course. The linearization allows to decouple the originally three-dimensional geometry into two planar engagements in perpendicular planes [3]. The X axis of the planar coordinate system is aligned with the initial line of sight. The assumption of constant velocities further simplifies the mathematical model. Indeed, since this phase of the engagement is dominated by intense lateral accelerations and by the dynamic response of the guidance system, the relatively small longitudinal accelerations have only a secondery and rather insignificant effect on the outcome. Moreover, the assumptions of an instantaneous evader dynamics and a near to "tail-chase" (or "head-on") geometry present the worst case for the pursuer.

The simplest mathematical model of an "end-game" for a meaningfull missile guidance analysis, with the geometry depicted in Fig 4., assumes first-order pursuer dynamics and takes into account the physical limits of the respective lateral accelerations. The equations of motion for a "tail-chase" are:

$$\dot{R} \approx \dot{x} = V_E \cos\gamma_E - V_P \cos\gamma_P \approx V_E - V_P = -V_c = const. \tag{35}$$

$$\dot{y} = V_y \approx V_E\gamma_E - V_P\gamma_P \tag{36}$$

$$\dot{V}_y = V_E\dot{\gamma}_E - V_P\dot{\gamma}_P = A_E - A_P \tag{37}$$

with

$$A_E = A_E{}^c = (V_E\Gamma_E)u_E; \qquad |u_E| \le 1 \tag{38}$$

$$\tau_P\dot{A}_P + A_P = A_P{}^c = (V_P\Gamma_P)u_P; \qquad |u_P| \le 1 \tag{39}$$

Equation (35) can be directly integrated, yielding

$$x(t) = V_c(t_f - t) = V_c t_{go} \tag{40}$$

where

$$t_f = \frac{R_0}{V_c} \tag{41}$$

is the fixed final time of the game.

4.2. Perfect Information "End-Game.

Such an "end-game" was first solved, assuming *perfect information*, in [27]. The cost function of the game is the the "miss distance", written for the linearized game geometry as

$$J = |y(t_f)| \tag{42}$$

to be minimized by the pursuer and maximized by the evader.

The above formulated zero-sum game solution is based on using a normalized time-scale

$$\theta = \frac{t_{go}}{\tau_P} \tag{43}$$

and the notion of *predicted zero effort miss distance*

$$Z = y + \tau_P \theta \dot{y} - {\tau_P}^2 \psi(\theta) A_P \tag{44}$$

where in last term

$$\psi(\theta) = \theta + e^{-\theta} - 1 \tag{45}$$

expresses the effect of non ideal guidance dynamics.

The game is characterized by a single parameter

$$\mu = \frac{A_{P\,max}}{A_{E\,max}} = \frac{V_P \Gamma_P}{V_E \Gamma_E} \tag{46}$$

The solution consists of partitioning the (θ, Z) game space into two regions of different solutions as depicted in Fig. 5. Inside the *minimal tube* $\mathcal{D}^o$, for $\theta \geq \theta_s$, where θ_s is the solution of

$$\theta_s = \mu \psi(\theta_s) \tag{47}$$

the optimal strategies are arbitrary and the Value of the game is constant.

$$C_m = A_{E\,max} {\tau_P}^2 [\theta_s - 0.5{\theta_s}^2(\mu - 1)] \tag{48}$$

Outside this region the optimal strategies are

$$u_P{}^* = u_E{}^* = sign Z \tag{49}$$

and the Value of the game is a uniquely determined smooth function of the state (θ, Z).

In the region of arbitrary strategy a linear guidance law, optimizing some secondary criteria, can be used. A time-varying gain yielding an infinite value at the boundary of the *minimal tube* is an other alternative.

The solution was extended for three-dimensional geometry [28-30] and for non ideal evader dynamics [29], where Eq. (38) is replaced by

$$\tau_E \dot{A}_E + A_E = A_E{}^c = (V_E \Gamma_E) u_E; \qquad |u_E| \leq 1 \tag{50}$$

This formulation assumes that the actual evader acceleration A_E is accurately mesured without any delay. If in this case the inequality

$$\mu \geq \frac{\tau_P}{\tau_E} \tag{51}$$

is satisfied, then θ_s and consequently C_m are zero. Anyhow, even for $\tau_E = 0$, the missile can guarantee (for $\mu \geq 2$) very small miss distances. A comparison with optimal control guidance laws [31] demonstrated the advantage of the differential game formulation and confirmed the robustness of the corresponding guidance law with respect to any target maneuver.

4.3. Imperfect Information "End-Game".

Until now the very attractive guidance law of 4.2 has not been implemented. One of the reasons is, undoubtedly, the non valid assumption of *perfect information.* In a realistic, thus noise corrupted, environment the implementation of this *perfect information guidance law* requires the estimation of the state. This is particularly important for a radar guided missile. For the estimation process the knowledge of the target maneuver structure and its eventual statistics is needed. Since an accurate estimation of the state variables is a prerequisite for an accurate guidance, the performance achieved by a *perfect information guidance law* using a non accurate *estimated state* is far from being satisfactory. From these observation it follows that one of the objectives of the evader in a noisy environment is to maximize the estimation error of the pursuer. A random target maneuvering denies all a-priori information and as a consequence can make the estimation process very unaccurate. Note, that an artificially generated random target motion by electronic countermeasures (ECM) has a similar effect. Therefore one can conclude, that in a noise corrupted scenario the optimal strategy of the evader must be different from the one predicted by Eq.(49). It has to be of a random nature, which is called in terms of game theory a "mixed strategy". Such a conclusion was reached in previous studies more than two decades ago [32,33], but its implication on the pursuers' optimal strategy (i. e. for missile guidance) has not been made clear. One has to remember that if in a game the optimal strategy of one player is "mixed", then (by definition) *the game has no solutions in pure strategies* and therefore the optimal strategy of the other player can be also a "mixed" one. (A "mixed strategy" is a probability distribution on a set of pure strategies. Each element of this set is a mapping of the available information set into an admissible control set.)

A recent investigation effort, sponsored by AFOSR Grant 86-0355, was the first one that proposed to consider a missile guidance based on a "mixed strategy" concept [34-38]. It formulated the terminal phase of the encounter between a radar guided missile and a highly maneuverable aircraft, which can also employ ECM, as an imperfect information zero-sum game played by the missile designer and the pilot of the target aircraft. The imperfect information is characterized by noisy and/or partial measurments of the state variables by the missile and by no measurments at all by the aircraft. The only information available to the pilot is a "warning" on missile lock-on.

The admissible controls of the evader are the lateral acceleration of the aircraft as given in Eq. (38) and the eventual use of ECM in the form of an "electronic jinking" [39]. For sake of simplicity it has been assumed that the elements δ_{E_i} of the evader's *pure strategy set* Δ_E (i.e. the maneuver and ECM options) are countable.

The missile designer faces the problem to find the *optimal pure strategy set* $\Delta_P{}^*$ of the pursuer, i. e. the number and the form of the guidance laws (each representing a *pure strategy* of the pursuer δ_{P_j}), to be programmed into the missile in order to achieve the best performance.

As the cost function of the game the single shot kill probability (*SSKP*) of the missile, a nonlinear function of the miss distance, was selected rather than the miss distance itself. In mathematical terms

$$J = SSKP = E\left(P_k[R(t_f)]\right) \tag{52}$$

where $P_k[.]$ is a real valued function, subject to $0 \leq P_k[.] \leq 1$. The expectation is taken over the ensemble of all noisy measurments and possible random target maneuvers. This cost function is to be maximized by the pursuer and minimized by the evader.

As a consequence of the short duration of "end-game" the *rules of the game* are such, that each player has to select at the outset of each encounter one of the elements from his *pure stratery set* to be used during the entire engagement.

The *rules of the game* and the assumption that the *pure strategy sets* are countable transform the the extremely complex original stochastic differntial game to a *matrix game*. The entries of this matrix $P_{i,j}$ are the outcomes of the respective encounters (each avaraged over a large number of samples) using the strategies δ_{E_i} and δ_{P_j}. Since matrix games are solvable by well known linear programming methods, one can compute without difficulties a "mixed strategy" solution.

After demonstrating by some simple examples that in certain circumstances a "mixed strategy" performs better than any of the available *pure* stategies [34], a game theoretical study was initiated in order to provide a rigorous mathematical frame [35] for further analysis. This study resulted in an iterarative algorithm for constructing a finite approximation of the *optimal pure strategy set* of the pursuer against any given *pure strategy set* of the evader.

In [36-38] some examples of guidance law synthesis for a skid-to-turn missile, based on the "mixed strategy" approach are presented. In these examples the *pure strategy set* of the evader is assumed to be a set of periodic, random phase, maximum turn maneuvers at prescribed frequencies in the range of $0 \leq \omega_E \leq 4rad/sec$, combined with an eventual "electronic jinking" of the similar range. The structure of the pursuer's *optimal pure strategy* is limited, for the sake of practical considerations, to the combination of the optimal pursuer strategy for a *prefect informaton* scenario, as proposed in [27], with an estimator in the form of a steady-state Kalman filter of a given structure. The guidance law synthesis has been oriented in these examples to search for the best parameters of the estimator. Both planar and three-dimensional engagement geometries were examined. For details the interested reader is refered to the original reports.

In spite of the limited freedom for guidance law optimization, the results are very encouraging. For an ECM-free planar scenario a single optimal guidance law (a particular case of a *pure strategy set* with a single element) was found, which performed much better against a randomly maneuvering target than any other guidance law previously discussed in the literature, e.g.[40], as it is shown in Fig. 6. This figure demonstrates the most important property of a guidance law derived by the "mixed strategy" approach (even if it turns out to be a *pure* strategy). This property is the absence of *weak spots* or in other words, the inherent *robustness* with respect to the unpredictability of the target maneuver.

The importance of a *true* "mixed strategy" for the missile is demonstrated in a two-dimensional ECM scenario [38]. Here the best *pure* guidance strategy resulted in *SSKP*=0.31, while a "mixed strategy" of two different guidance laws yields *SSKP*=0.4, both for a rather small warhead with the lethal range of 3 meters. This is an improvement of more than 30%, equivalent to a similar reduction in the number of missiles needed for the same outcome.

It is important to note that warhead lethality, expressed by some nominal lethal renge, is a sensitive parameter of any guided missile design. It directly effects the optimal guidance law parameters, not to speak about the resulting *SSKP*. For example in a three-dimensional ECM scenario the value of the *SSKP* raised from 0.43 for a 3 meter warhead, to 0.69 for a 4 meter lethal radius. This (more than 60%) improvement is much higher than would be expected due to the 33% increase of the lethal range.

5. CONCLUSIONS.

During the last three decades the technology of interceptor missiles has made an impressive progress, but unfortunately guidance law synthesis has remained conservative. The guidance problem of an interceptor missile is, by definition, a pursuit-evasion differential game. This formulation, however, has not yet been used in any known missile system or in any ongoing design.

In the present paper new guidance concepts, – such as the *effective firing envelope* or "mixed strategies",– based on a differential game formulation, are introduced and their potential impact on future missile design is demonstrated. The challanges of maximizing the *effective firing envelope* ("capture zone") of a short-range missile and the terminal guidance ("end-game") of interceptor missiles are analysed by different dynamic models. For the "end-game" analysis both perfect and imperfect information structures are considered.

The optimal missile guidance laws, derived via differential game analysis are substantially different from those based on classical or optimal control concepts. The are characterized by an inherent *rubustness* against any possible target behavior, a consequence of the explicit consideration of optimal evasive maneuvering.

The present paper could not cover all the problems involved in interceptor missile design. The recent investigations reviewed in this paper serve merely as milestones in the direction of applying differential game theory for optimal missile guidance. Nevertheless, they have demonstrated that a guidance law synthesis based on differential game concepts can enhance missile performance by yielding enlarged *effective firing envelopes*, as well as improved homing accuracy.

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LIST OF FIGURES.

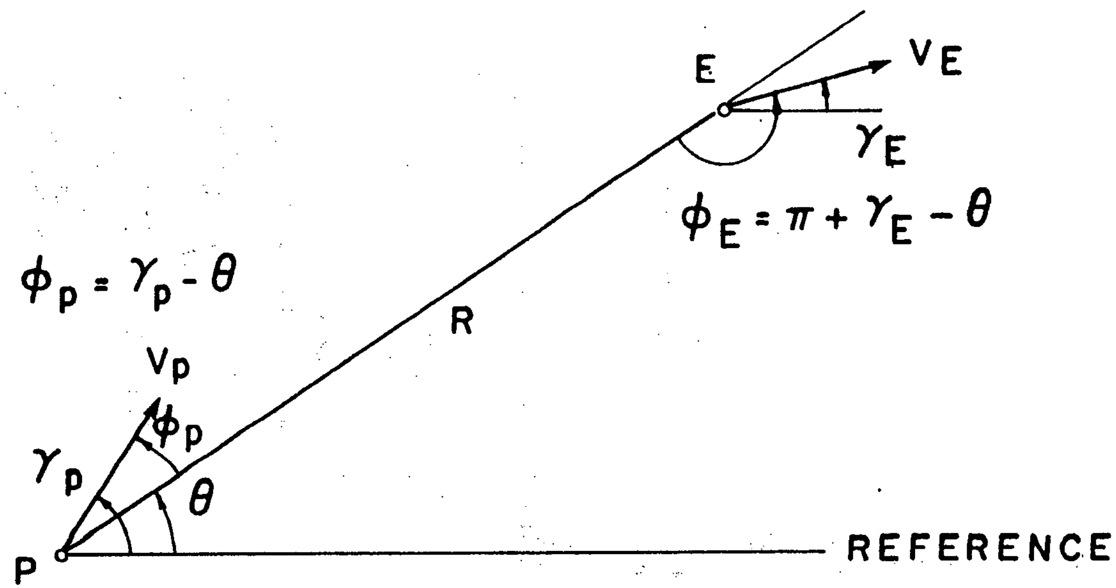

Fig. 1. Horizontal game geometry.

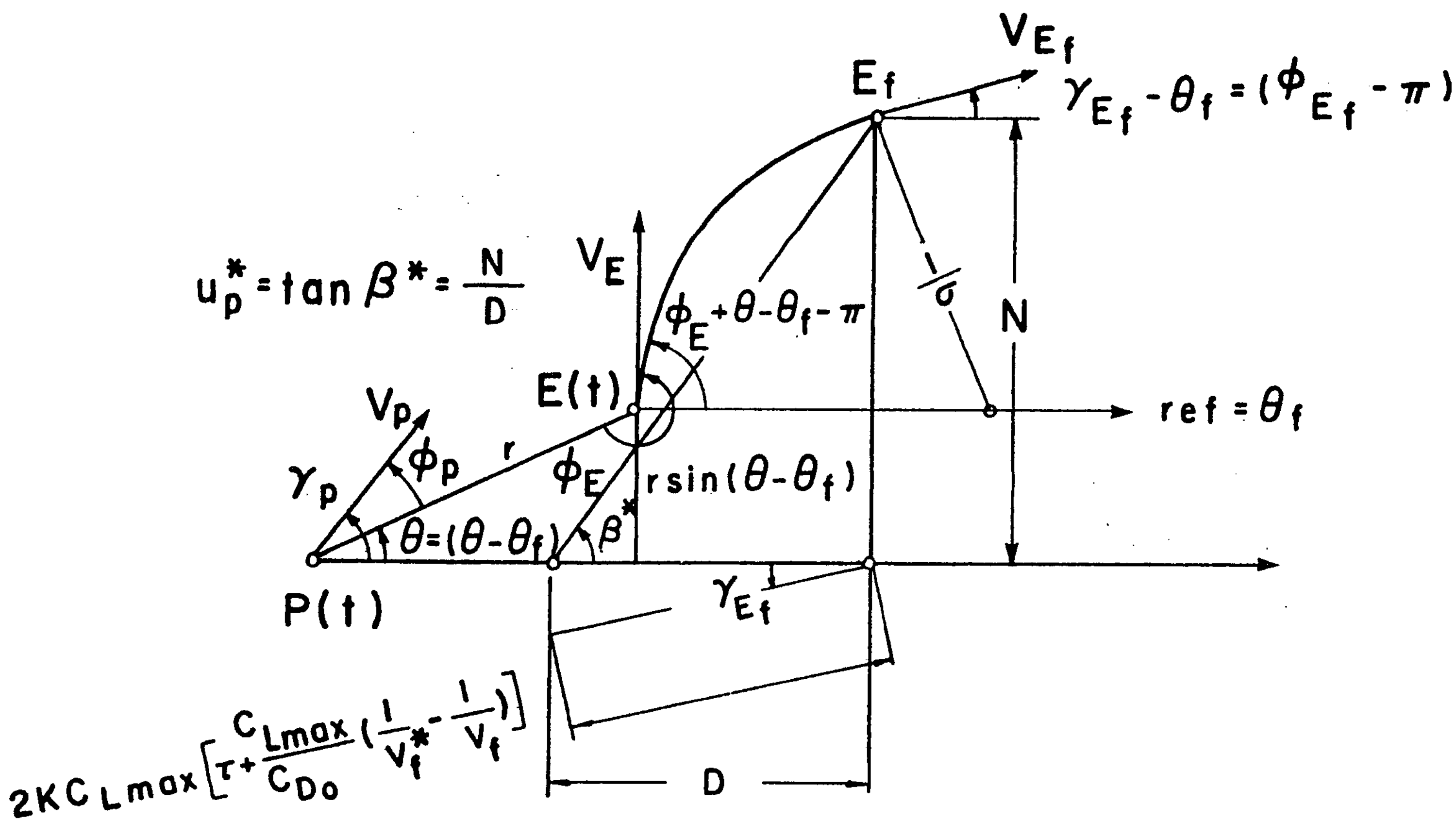

Fig. 2. Interpretation of optimal pursuer strategy.

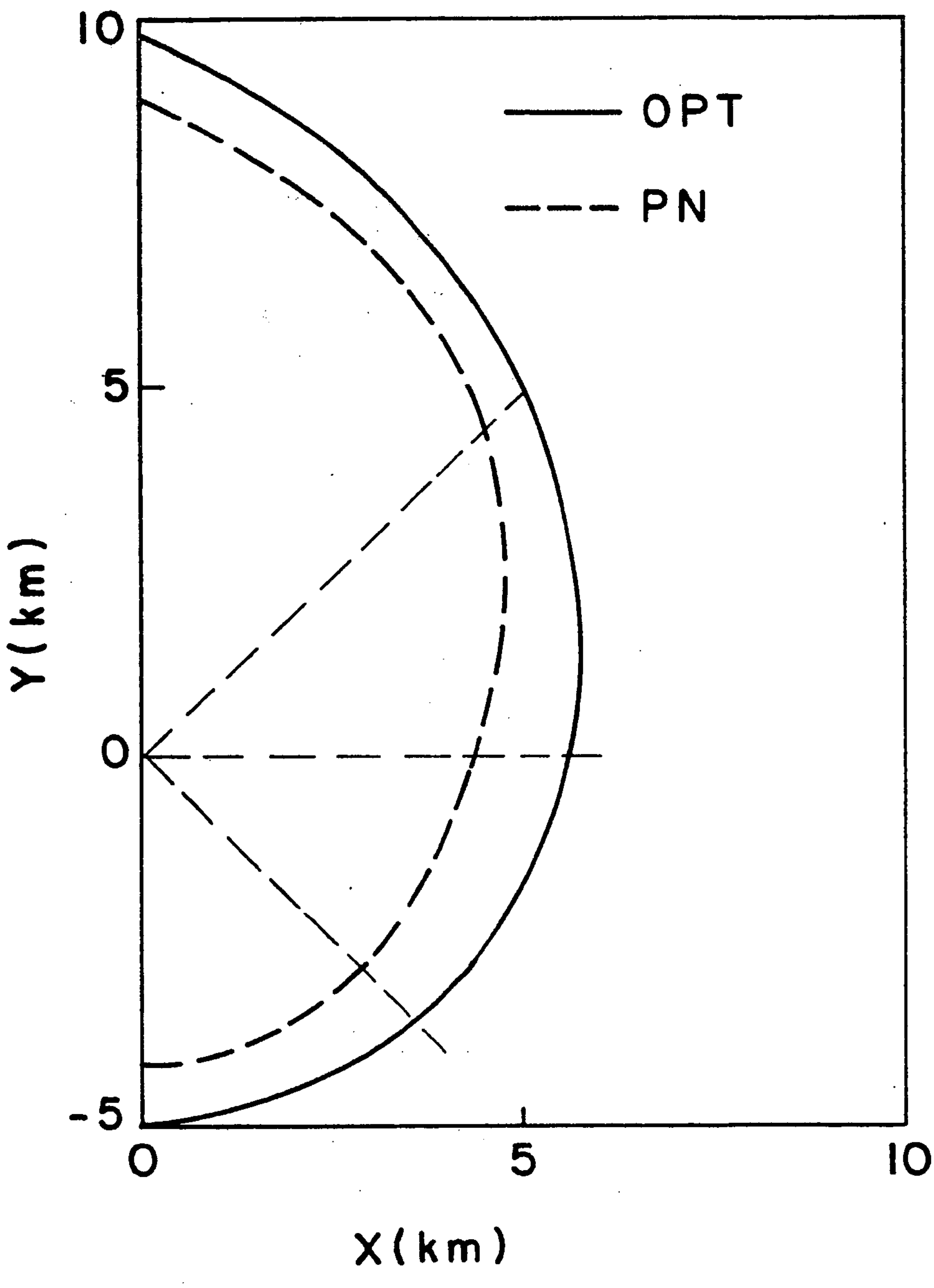

Fig. 3. Capture zone comparison.

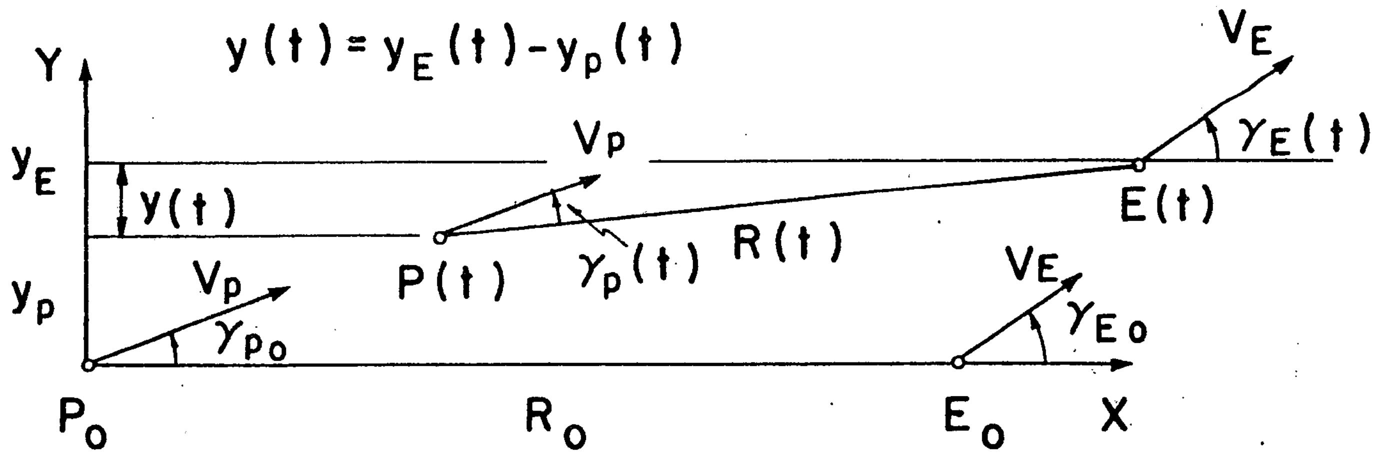

Fig. 4. Linearized "end-game" geometry.

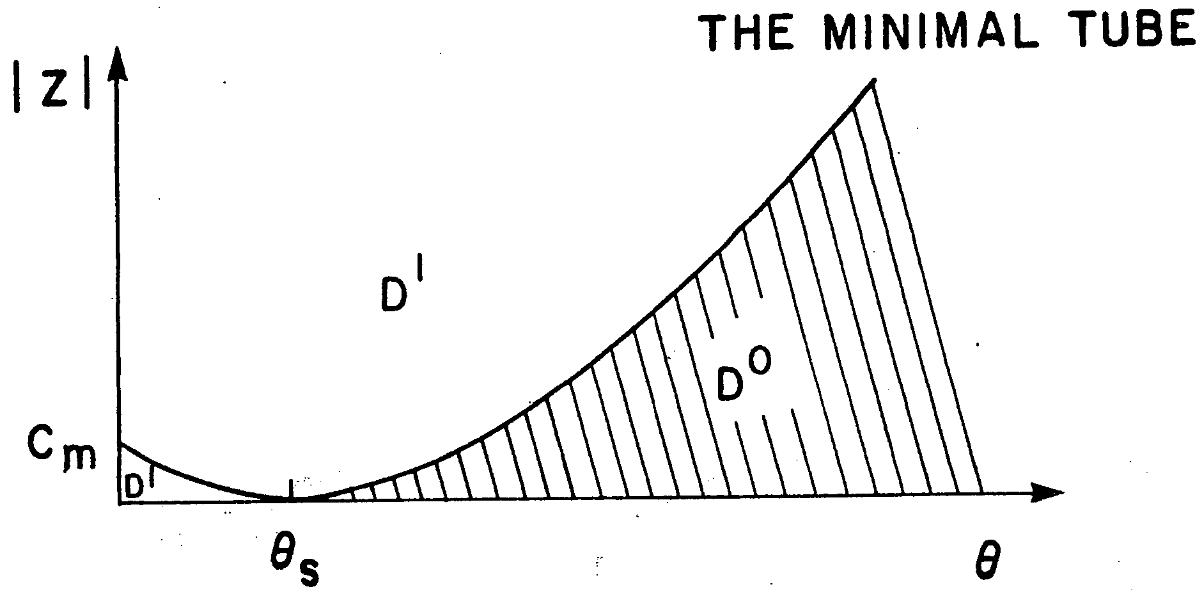

Fig. 5. Partition of the (θ, Z) game space.

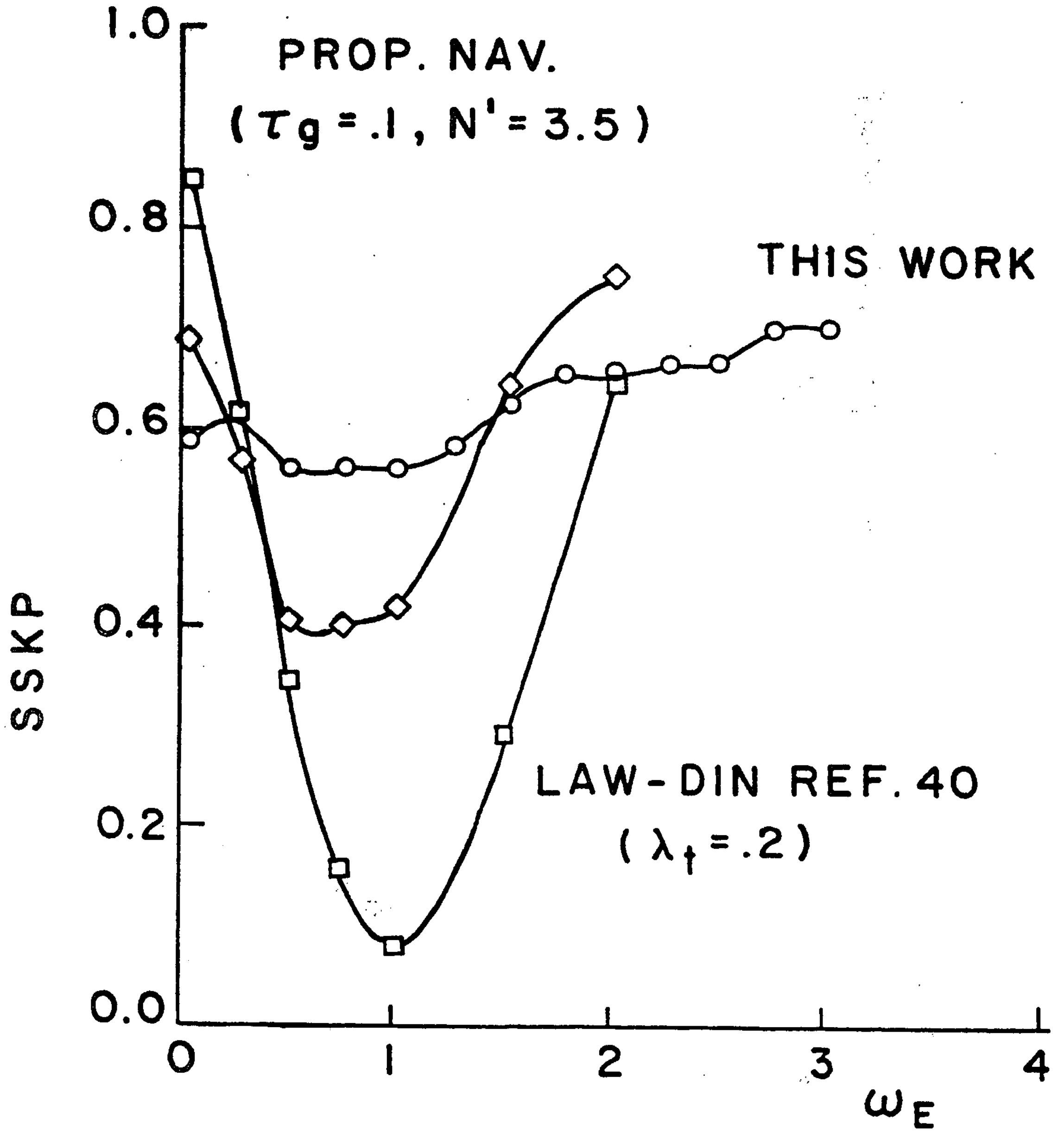

Fig. 6. Guidance law comparison in an ECM-free planar scenario.

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